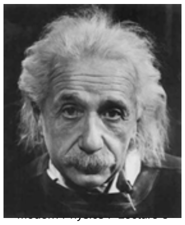
University of San Francisco

Modern Physics for Frommies I

Albert Einstein's Universe

Lecture 5



Agenda

- Administrative matters
  - Upcoming Physics Colloquia
    - 2nd handout
    - Wiki below
  - Progress on Blackboard® site
    - http://modernphysicsfrommies.wiki.usfca.edu
    - Hard copies ?
- Apparent Paradoxes
  - Twin paradox
  - Pole in the barn paradox
- Spacetime
  - Light cones
  - 4-D distances, a Lorentz invariant
  - The twins visit 4-space

Apparent Paradoxes

The twin paradox:

Identical twins, Pat and Mike, born at the same moment

At age 20: Pat travels to A-Centauri at a speed of 0.9*c* and then returns to Earth at the same speed.
Mike remains on Earth and works the family farm.

Pat returns to Earth to find that *8.4* years have passed.
i.e Mike is *28.4* years of age.

Pat however has only aged

$$t = t_p\gamma = 3.7 \text{ years}$$

i.e. Pat is *23.7* years of age.

OK, that's the picture in Mike's frame.

What happens in Pat's frame?

Pat will see himself at rest and Mike traveling at *0.9c*

Doesn't this mean that at the reunion their ages will be reversed? **PARADOX**

Symmetry in what the twins observe is only apparent .

Symmetry is broken by the out and back nature of Pat's journey.

Only Mike is always in a single inertial frame. Pat is in several

Proper application of Lorentz transforms → Pat is indeed younger

One can also invoke General Relativity since accelerations are involved.

Thornton/Rex, Modern Physics for Scientists and Engineers, 2/e
Table 2.1

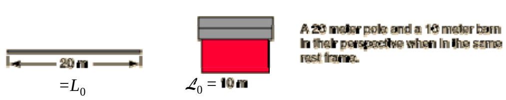
TABLE 2.1
Twin Paradox Analysis*

Item	Measured by Frank (remains on Earth)	Measured by Mary (traveling astronaut)
Time of total trip	$T = 2L/v$	$T' = 2L/\gamma v$
Total number of signals sent	$\nu T = 2\nu L/v$	$\nu T' = 2\nu L/\gamma v$
Frequency of signals received at beginning of trip ν'	$\nu\sqrt{\frac{1-\beta}{1+\beta}}$	$\nu\sqrt{\frac{1-\beta}{1+\beta}}$
Time of detecting Mary's turn-around	$t_1 = L/v + L/c$	$t'_1 = L/\gamma v$
Number of signals received at the rate ν'	$\nu' t_1 = \frac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu' t'_1 = \frac{\nu L}{v}(1-\beta)$
Time for remainder of trip	$t_2 = L/v - L/c$	$t'_2 = L/\gamma v$
Frequency of signals received at end of trip ν''	$\nu\sqrt{\frac{1+\beta}{1-\beta}}$	$\nu\sqrt{\frac{1+\beta}{1-\beta}}$
Number of signals received at rate ν''	$\nu'' t_2 = \frac{\nu L}{v}\sqrt{1-\beta^2}$	$\nu'' t'_2 = \frac{\nu L}{v}(1+\beta)$
Total number of signals received	$2\nu L/\gamma v$	$2\nu L/v$
Conclusion as to other twin's measure of time taken	$T' = 2L/\gamma v$	$T = 2L/v$

*After A. French, *Special Relativity*. New York: W. W. Norton, 1968, p. 158.

The Pole in the Barn Paradox:

Farmer Brown wants to store a pole in his barn but the pole is too long.

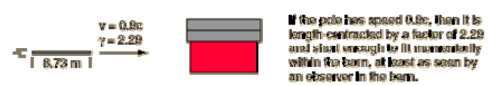


He hires an out of work runner (disqualified for use of banned substances) to carry the pole into the barn at a speed of 0.9*c*

$$\beta = \frac{v}{c} = 0.9$$

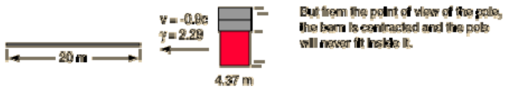
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.29$$

In the barn's (or the farmer's) frame, $L = L_0 / \gamma$ = 8.73 m, pole should momentarily fit.



This is the barn frame.

In the runner's frame, the barn appears to have shrunk to $\mathcal{L} = \mathcal{L}_0 / \gamma$ = 4.37 m, the pole should stick out even worse than before



PARADOX

Does it fit or not?

Q. What is meant by "the pole is in the barn"?

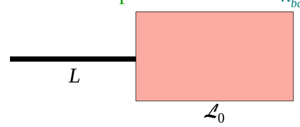
A. Momentarily, both the front and the back doors of the barn can be closed containing the entire pole.

The closing of the doors, if simultaneous in one frame, need not be so in another frame.

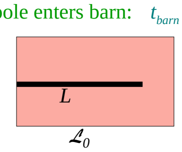
Let's consider events involving the two ends of the pole.

<u>Barn frame</u>:

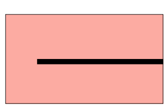
Front of contracted pole enters barn: $x_{barn} = 0,\ t_{barn} = 0$



Back of contracted pole enters barn: $t_{barn} = \dfrac{8.73\text{ m}}{0.9c} = 32.29\text{ ns}$



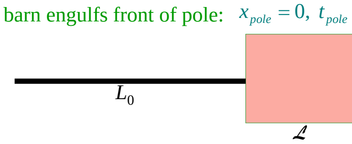
Front of contracted pole leaves barn: $t_{barn} = \dfrac{10\text{ m}}{0.9c} = 37.04\text{ ns}$



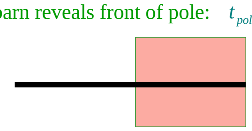
Back of pole enters barn before front leaves. Doors can be simultaneously shut momentarily. Let's quickly close and open the doors at t_{barn}= 32.35 ns

<u>Pole frame</u>:

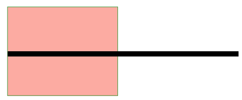
Front of contracted barn engulfs front of pole: $x_{pole} = 0,\ t_{pole} = 0$



Back of contracted barn reveals front of pole: $t_{pole} = \dfrac{4.37\text{ m}}{0.9c} = 16.4\text{ ns}$



Front of contracted barn engulfs back of pole: $t_{pole} = \dfrac{20\text{ m}}{0.9c} = 74.07\text{ ns}$



Back of contracted barn reveals back of pole: $t_{pole} = 16.14\text{ ns} + 74.07\text{ ns} = 90.21\text{ ns}$

In the barn frame both doors close and immediately reopen at t_{barn}= 32.35 ns.

Let's Lorentz transform to the pole frame.

Front door: $t_{pole} = \gamma\left(t_{barn} - \dfrac{vx_{barn}}{c^2}\right) = 2.29\left(32.35\text{ ns} + 0\right) = 74.08\text{ ns}$

Back door: $t_{pole} = \gamma\left(t_{barn} - \dfrac{vx_{barn}}{c^2}\right) = 2.29\left(32.35\text{ ns} - \dfrac{(0.9c)(10\text{m})}{c^2}\right)$

$= 5.38\text{ ns}$

Doors closing-opening are simultaneous in the barn frame but not in the pole frame.

From the pole point of view the front door closes and opens just after it passes the back of the pole. The back door closes and opens earlier before the front of the pole is reached. The doors closings and openings are not simultaneous and allow the passage of the barn over the pole without hitting either door

So what finally happens? New element. What happens as the pole is brought to stop?

Suppose the farmer grabs the back end of the pole as it enters the barn and brings the pole to a stop.

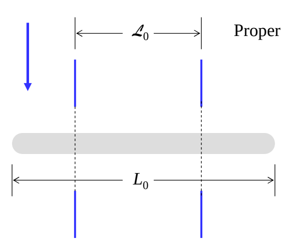
The pole must then revert to its proper length in the barn frame. Either the door is knocked back open, a hole is punched in the far wall of the barn or both.

Farmer Brown is not happy!

$K = mc^2(\gamma - 1) = (25\text{ kg})(3\times10^8\text{ m/s})^2(2.29-1) = 3\times10^{18}\text{ J}$

Hiroshima, August 1945, *Little Boy*, ≈ 15 kilotons $= 6.3\times10^{13}$ J

The Relativistic Butcher Shop:



Proper length between blades: $\mathcal{L}_0 = 15$ cm

Salami proper length: $L_0 = 30$ cm

Call this is the shop frame or the blade frame. If the salami is at rest in this frame and the blades drop simultaneously, the salami will be cut into three pieces.

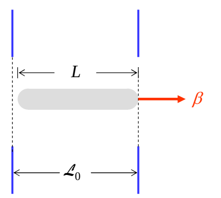
Now give the salami a velocity of 0.9c w.r.t. to the shop and blades.

$$\beta = \frac{v}{c} = 0.9$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.29$$

Moving salami in shop frame

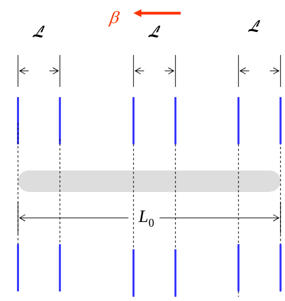
Salami Lorentz contracts to $L = L_0/\gamma = 13$ cm in this frame



Salami is not cut

Now, let's go to the rest frame of the salami and observe the blades moving at 0.9c.

Now, the blade separation appears contracted to $\mathcal{L} = \mathcal{L}_0/\gamma = 6.5$ cm.

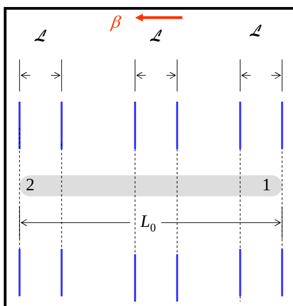


The salami always gets cut

PARADOX

Not so fast, we have shown the blades dropping simultaneously, just as in the shop frame. As relativity mavens, we know that simultaneity is a frame dependent.

(TO BE CONTINUED)



Salami frame:

End 1 is x_s =0, end 2 is x_s = 30 cm

Left blade passes over end 1 at $t_s = x_s = 0$

Right blade passes over end 1 at

$$t_s = \frac{6.5 \text{ cm}}{0.9c} = 0.22 \text{ ns}$$

4-D Minkowski Spacetime

Recall the matrix formulation of the Lorentz transform for a boost along the (+) x-axis

$$ct' \leftarrow \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\left(\frac{-v}{c^2}\right) & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \rightarrow ct'$$

An event location is noted as a 4-D vector with the 4th dimension being time.

To keep the same units for all axes replace t with ct.

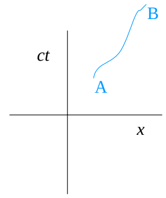
Hermann Minkowski

1864-1909

"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

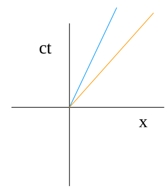
Hermann Minkowski in his talk at the 80th Assembly of German Natural Scientists and Physicians, September 21, 1908

Minkowski space and Minkowski diagrams



Events A and B are connected by a trajectory called a *worldline*

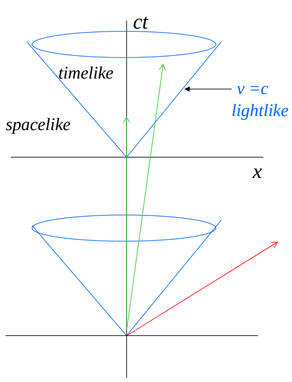
Worldlines are straight in inertial frames.



-Spaceship leaves origin w. velocity *v*. Slope=*c/v*

- Light signal velocity *c*

Light cone

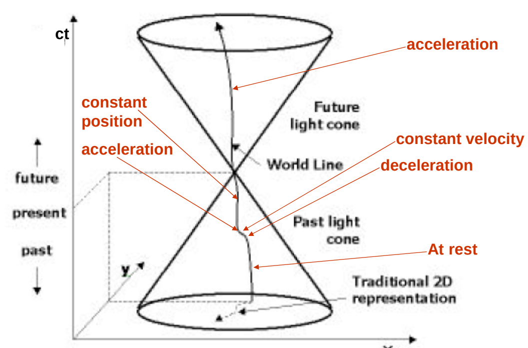


Points or locations are events.

Events move on paths called world lines

Only timelike world lines are allowed

Cannot see outside light cone



Lorentz Invariants and 4-dimensional distance: *interval* (s)

In 3-D: $d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is the square of the distance between 2 points. $d = 0$ only if the two points are colocated.

Galilean transforms $\Rightarrow d^2$ and hence d are the same in any inertial frame and are thus said to be *invariant.*

Is there a quantity, similar to d^2, which is invariant under the Lorentz transforms in 4-D?

Consider two inertial frames, F and F', and the quantities

$$s^2 = x^2 - (ct)^2$$

$$s'^2 = x'^2 - (ct')^2$$

Applying the Lorentz transforms we find that $s^2 = s'^2$, so the quantity s^2 is an invariant.

Including the other spatial coordinates, y and z, we have $s^2 = x^2 + y^2 + z^2 - (ct)^2$ as an invariant.

In analogy to the distance between two points in 3-D we can determine the 4-D separation of two events in Minkowski space.

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (z)^2 - (\Delta ct)^2$$

Δs is called the *spacetime interval* between events

Applying the Lorentz transforms we find that $s^2 = s'^2$, so the quantity s^2 is an invariant.

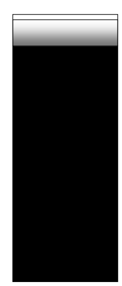
Including the other spatial coordinates, x and y, we have $s^2 = x^2 + y^2 + z^2 - (ct)^2$ as an invariant.

s is the time interval experienced by a clock moving between 2 events. (proper time)

s = 0 for any two points lying on the light cone

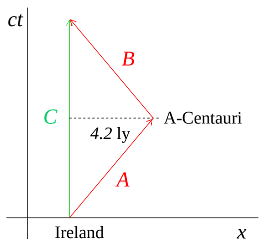
Meanwhile, back at the "ranch", actually the farm,

Pat and Mike are still trying to figure things out.



Guinness is good for you!

Pat and Mike visit 4-space:



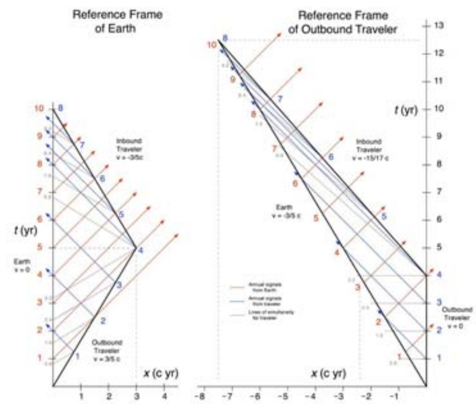
In 3-D we have the triangle inequality

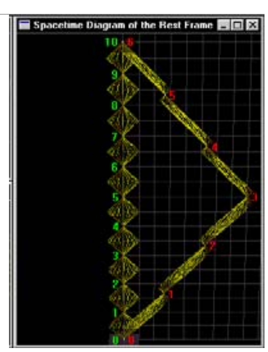
$$A + B \geq C$$

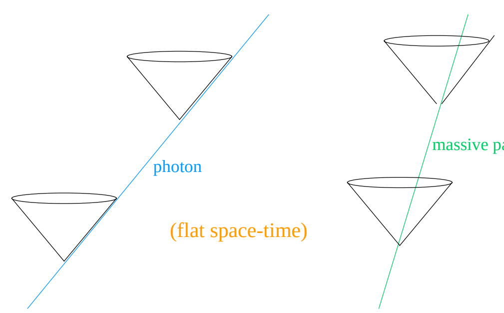
In our 4-D space this Inequality is reversed due to our definition of the interval. So

$$A + B \leq C$$

The traveling twin returns the younger man. Poor Mike!

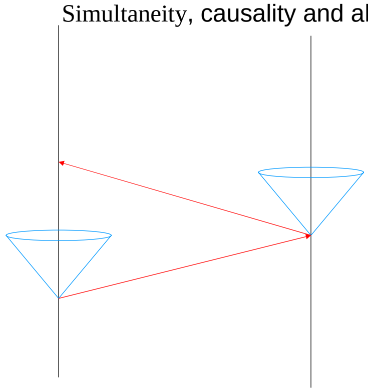




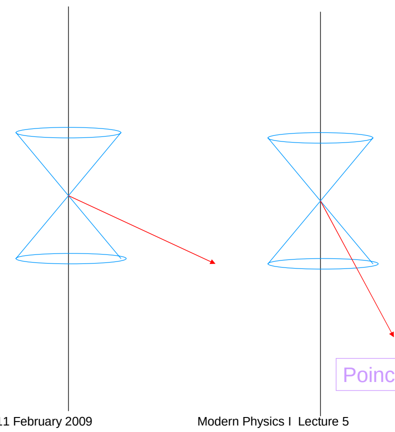


The family of light cones at all points forms the Minkowskian geometry of space-time.

Simultaneity, causality and all that redux



11 February 2009 Modern Physics I Lecture 5 33



Do diagrams like these mean anything at all?

I think not, but perhaps I am just ignorant.

Poincaré motion

11 February 2009 Modern Physics I Lecture 5 34

Material bodies cannot travel faster than *c* =>
world lines must lie within the light cones

World lines of photons lie along the light cones

N particle is permitted to follow a world line lying outside the light cone.

More generally, no signal is permitted to travel outside the light cone.

11 February 2009 Modern Physics I Lecture 5 35