RECAP HANDOUT II

DEFINITIONS: Let A and B be events in a probability space S. (1) If B occurs, then the *CONDITIONAL PROBABILITY OF A GIVEN B* is $P(A|B) = P(A \cap B) / P(B)$. **(2) If $P(A|B)=P(A)$, then we say that *A is independent of B.* Note that in this case (i) $P(A \cap B)=P(A)P(B)$ and also (ii) B is independent of A, i.e., $P(B|A)=P(B)$. (3) If A1,A2,... is a sequence of independent events (that is, any two of them are independent), then $P(A1 \cap A2 \cap \ldots)=P(A1)P(A2)\ldots$**

EXAMPLE: S=the uniform space of outcomes from rolling a pair of dice, A=rolling a five, B=rolling a 10, and C=one die shows a 2. THEN P(A)=1/9, P(B)=1/12, P(C)=11/36, P(A|C)=2/11, and P(B|C)=0.

WAITING FOR AN EVENT TO HAPPEN FOR THE FIRST TIME.

Let us consider the sequence of independent trials of tossing a coin with probability p>0 of a heads and probability q=1-p of a tails. A sequence of independent trails each with only two possible outcomes, p>0 and q=1-p, is called a sequence of *BERNOULLI TRIALS.* [Johann Bernoulli (1667-1748).] Now for each natural number n, let Tn=tails on the nth toss, Hn=heads on the nth toss, and An=heads appears for the first time on the nth toss. What is P(An)? If heads first occurs at toss n, then $An=T1 \cap T2 \cap \ldots \cap T(n-1) \cap Hn$ so $P(An)=q^{n-1}p$.

APPLICATION: Let H=at least one toss is heads. What is P(H)? Now H is the union of A1 and A2 and A3... and all of these events are disjoint (if the first heads is on the nth toss, then it cannot first appear on the mth toss). Therefore,

$$P(H)=P(A1)+P(A2)+\ldots+P(An)+\ldots$$
$$= p \quad + qp \quad +\ldots+ q^{n-1}p +\ldots$$

This infinite sum is called a *GEOMETRIC SERIES* and its sum S is given by

$$S = p/(1-q)$$

In this case, since p=1-q, we have P(H)=1. Thus, it is with certainty (probability 1) that at least one toss results in a heads.

MONKEY AT THE TYPEWRITER: Suppose that there are N keys on a typewriter and a monkey continually hits the keys at random with the probability of any one key being hit of 1/N. Each hit is an independent trial and each string of, say, M trials either spells out the complete works of Shakespeare with probability $p=(1/N)^M>0$ or not with probability q=1-p. This is a sequence of Bernoulli trials and, as we saw above, if H=at least one string of M trials is the complete works of Shakespeare, then P(H)=1.

RARE VS EXTRAORDINARY EVENTS: Toss a fair coin 10 times and observe the outcomes {h h t h t t h t h t}. Now toss it 10 more times and observe {h h h h h h h h h h}. Which set of outcomes is more rare? Which is more extraordinary?

(over)

Counting: Recall the COUNTING PRINCIPAL

Examples: (1) Count the number of ways that three people can line up for a photo. Each way is called a *permutation* and the product 3x2x1 is called 3 *factorial* written 3!. In general, for any counting number n, we define n! = n(n-1)(n-2)…(3)(2)(1) and also 0!=1.

(2) Count the number of ways that three people out of five can line up. Notice that 5x4x3=5!/(5-3)!

(3)Count the number of ways that a committee of three people can be chosen from a group of 5. Notice that 5x4x3/3x2x1 = 5!/3!(5-3)!. This is called a *combination* (we often say "5 choose 3").

Guidelines for Choosing a Counting Method.

(1) If the items to be selected can be repeated, use CP. Example: Roll the dice 5 times.

(2) If selected items cannot be repeated and order is important, use permutations; r items can be selected from n objects in n!/(n-r)! ways. Example: Elect a Pres., a VP and a Treas. in a club of 10 people.

(3) If selected items cannot be repeated and order is not important, use combinations; r items can be selected from n objects without regard to order in n!/r!(n-r)! ways. Example: Pick a winner in the California Lotto. Answer: Pick 5 numbers without regard to order from 47 and then pick one *mega* from 27 numbers so (47 choose 5)x27 = [47!/5!42!]x27=41,416,353 ways. Good Luck!