

### Lecture 3. Bayes' Theorem (Thomas Bayes 1702-1761)

**Problem 1.** Roll a pair of (fair) dice. (a) If the sum of the numbers rolled is odd, what is the probability that at least one of the numbers is 6? (b) If at least one of the numbers is 6, what is the probability that the sum is odd?

**Problem 2.** A patient suffers from a condition which is fatal 50% of the time but a possible treatment involves surgery. Research shows that 40% of survivors and 10% of non-survivors had surgery. Would you opt for the surgery? If so, what is the probability of your survival?

**Problem 3.** Toss a (fair) coin four times. If the fourth toss is a head, what is the probability that the first three were also heads?

#### TOPICS

**A.** Conditional Probability: Given that A and B are events in a probability space S with  $p(B) > 0$ , then the *conditional probability* of A given that B has occurred is written  $p(A|B)$  and defined by

$$P(A|B) = p(A \cap B) / p(B).$$

We say that the events A and B are *independent* if  $p(A|B) = p(A)$ . Note that in this case  $p(A \cap B) = p(A)p(B)$  and that  $p(A|B) = p(A|B^c)$ .

**B.** Bayes' Theorem

$$P(A|B) = [P(B|A) \cdot P(A)] / [P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)]$$

Problems 1, 2 and 3.