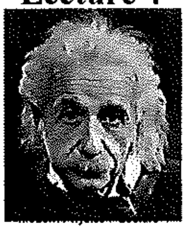
# University of San Francisco

# Modern Physics for Frommies I

## *Albert Einstein's Universe*

## Lecture 4





## Agenda

- Administrative matters
  - Upcoming Physics Colloquia
  - Progress on Blackboard® site
- Lorentz Contraction
- Velocity Addition
- Examples, Experiments and Observations
- Movie (?)
- Doppler Effect
- Čerenkov Radiation
- Apparent Paradoxes
- Spacetime



## Physics Department Colloquia

<u>Candidates for astrophysics faculty position:</u>

~~Monday, February 2, 4:00-5:00 PM in HR 232 – Kartik Sheth, Cal Tech. *Tracing the Evolution of Galaxies from Our Local Neighborhood to the Cosmic Nursery.*~~

Wednesday, February 4, 4:00-5:00 PM in HR 127 – Brenda Frye, City University of Dublin. *Conditions and Environments of Galaxies at Early Times.*

Monday, February 9, 4:00-5:00 PM in HR 232 – Nate McCrady, UCLA, *Young Massive Clusters: Building Blocks of Galaxies.*

Regular colloquium series restarts February 25. Speakers and topics TBA.



**Lorentz transforms (frame boost in *x*) are:**

| | | |
|---|---|---|
| $x' = \gamma(x - vt)$ | And the inverses | $x = \gamma(x' + vt)$ |
| $y' = y$ | | $y = y'$ |
| $z' = z$ | | $z = z'$ |
| $t' = \gamma(t - vx/c^2)$ | | $t = \gamma(t' + vx'/c^2)$ |

Time Dilation

Gedanken experiment → $\Delta t = \gamma \Delta t_0$

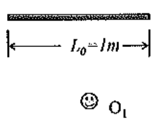
Lorentz transforms → $\Delta t = \gamma \Delta t_0$

## Length Contraction

The length of an object is measured to be shorter when it is moving w.r.t. an observer than when it is at rest.

Gedanken Experiment

Observer is at rest w.r.t meter stick.




$L_0$ is called the *proper length*

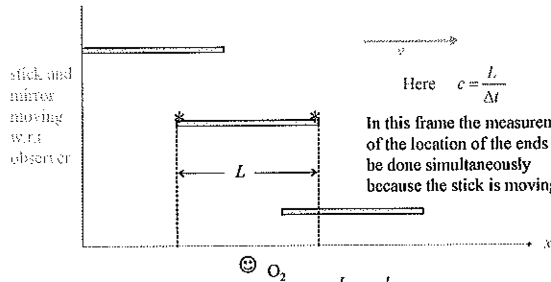
Time for light to traverse $L_0$ is $\Delta t_0$

$$c = \frac{L_0}{\Delta t_0}$$

Measurements of the locations of the 2 ends do not have to be made simultaneously since the stick is at rest w.r.t. $O_1$

Now, let's look at a 2nd observer with the stick passing at velocity $v$.






Here $c = \frac{L}{\Delta t}$

In this frame the measurement of the location of the ends must be done simultaneously because the stick is moving.

$c$ is the same in both frames, so: $c = \frac{L_0}{\Delta t_0} = \frac{L}{\Delta t}$ Apply time dilation

$$= \frac{L}{\gamma \Delta t_0}$$

Note that $\frac{(1/\gamma)L_0}{\gamma \Delta t_0} = \frac{L_0}{\Delta t_0} = c \Rightarrow L = \frac{1}{\gamma} L_0$



Lorentz Transform

$$x_0 = \gamma(x - vt) \Rightarrow L_0 = \gamma(L - v\Delta t)$$

In theLab frame $\Delta t = 0 \quad \Rightarrow L_0 = \gamma L$

$$L = \frac{1}{\gamma} L_0$$

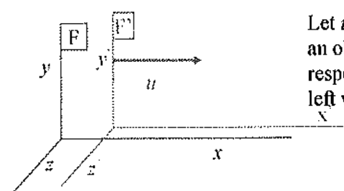
$$L = \frac{1}{\gamma} L_0 = \sqrt{1 - \frac{v^2}{c^2}} L_0$$ *Lorentz contraction*

Lorentz and Fitzgerald (1890's): Length contracts by a factor of $(1 - v^2/c^2)^{1/2}$ in the direction of motion through the ether, an electrical effect on intermolecular forces.

Rather than invoking an unknown electrical interaction with intermolecular forces we see the contraction arising from a coordinate transform under which the speed of light is invariant.



## Velocity Addition




Let $u$ and $u'$ be the velocities of an object in F and F' respectively. F' moves to the left with velocity $v$ w.r.t. F

In Galilean relativity we simply have $v' = v - u$

and

$$v = v' + u$$

This is not in agreement with the Lorentz transforms

$x' = \gamma(x - ut)$ And the inverses $x = \gamma(x' + ut)$

$y' = y$ $y = y'$

$z' = z$ $z = z'$

$t' = \gamma(t - ux/c^2)$ $t = \gamma(t' + ux'/c^2)$

$$v_{x'} = \frac{\Delta x'}{\Delta t'} \qquad v_x = \frac{\Delta x}{\Delta t}$$

Using differential calculus or else some messy algebra



$$v_{x'} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \qquad v_x = \frac{v_{x'} + u}{1 + \frac{uv_{x'}}{c^2}}$$

$$v_{y'} = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad v_y = \frac{v_{y'}}{\gamma\left(1 + \frac{uv_{x'}}{c^2}\right)}$$

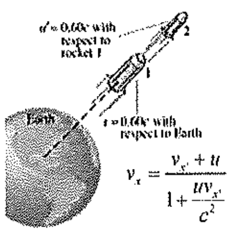
$$v_{z'} = \frac{v_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad v_z = \frac{v_{z'}}{\gamma\left(1 + \frac{uv_x}{c^2}\right)}$$

Note the effects transverse to the boost due to the Lorentz transformation of the time coordinate.

Again, for $v$ and $u << c$ these reduce to the Galilean velocity transforms



Example: Velocity Addition




Find speed of 2 w.r.t. Earth

Earth is frame F.
2 is frame F'. $x$, $x'$ are along flight

$$v_x = \frac{v_{x'} + u}{1 + \frac{uv_{x'}}{c^2}} = \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c$$

Galilean transform → $v_x = 1.20c$

There is no addition of velocities that will result in $v > c$

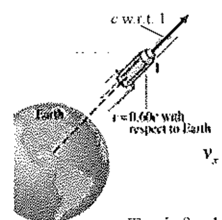
TRY IT



Example: Constancy of $c$

Replace 2 with light from 1's headlight in previous example

Find speed of light w.r.t. Earth




Earth is frame F.
1 is frame F'. $x$, $x'$ are along flight

$$v_x = \frac{v_{x'} + u}{1 + \frac{uv_{x'}}{c^2}} = \frac{c + 0.60c}{1 + \frac{(0.60c)c}{c^2}} = \frac{1.60c}{1.60} = c$$

Try it for 1 traveling at $0.99c$ w.r.t. Earth

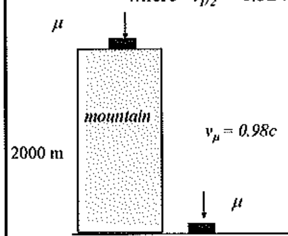
$$v_x = \frac{v_{x'} + u}{1 + \frac{uv_{x'}}{c^2}} = \frac{c + 0.99c}{1 + \frac{(0.99c)c}{c^2}} = \frac{1.99c}{1.99} = c$$

## Experiments and observations

Muon decay: $m_\mu \approx 200 m_e$ $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

Radioactive decay law: $N = N_0 e^{-(\ln 2)\left(\frac{t}{t_{1/2}}\right)}$

where $t_{1/2} = 1.52 \times 10^{-6}$ seconds




Count for some period of time

Top = 1000 ± 31.6 muons

Bottom = 540 ± 23.2 muons



Classically, a particle moving at $0.98c$ covers 2000 m in $6.8 \times 10^{-6}$ sec.

Decay law $\Rightarrow$ only 45 muons should survive the trip, yet we see 540.

Relativistically, we realize that the quoted half-life of $1.52 \times 10^{-6}$ sec. is that of a $\mu$ at rest. At $\beta = 0.98$, time dilation is significant.

An observer in the lab will perceive that a clock moving with the $\mu$ to be slowed by a factor of $\gamma$.

$\beta = 0.98 \Rightarrow \gamma = 5$

Decay law corrected $\Rightarrow$ 538 $\mu$ should survive the trip. Agreement with experiment.



**Alternatively, examine the problem from the point of view of an observer traveling with the $\mu$.**

This observer sees the 2000 m flight path as length contracted by a factor of $1/\gamma$ to 400 m.

The time to travel this contracted difference is thus reduced by a factor of 1/5.

Decay law corrected $\Rightarrow$ *538* surviving $\mu$s.

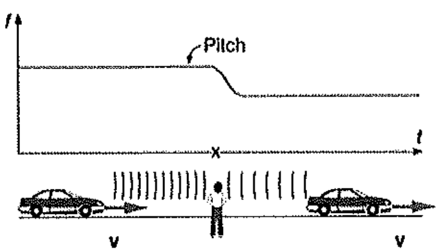
Identical result, in agreement with experiment, is obtained by using either time dilation or space contraction.



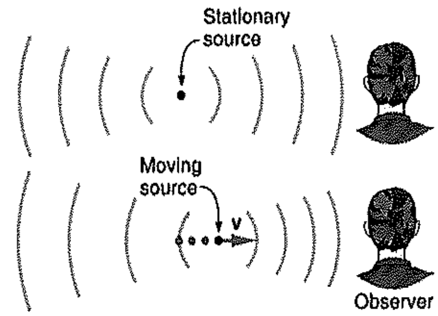
# MOVIE TIME

## Doppler Effect

Classically, e.g. sound waves






$n' = n\,[v/(v \pm v_s)]$

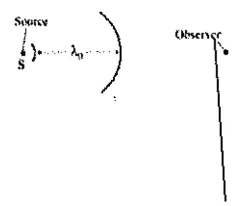
- $v$ = wave velocity
- $v_s$ = source velocity
- ($\pm$) = away/towards



For light we must treat the problem relativistically.

No medium (ether) so only relative velocity of source and observer matters.

We cannot have $v>c$.




Source at rest w.r.t. observer emitting light of frequency $f_0$ and wavelength $\lambda_0 = c/f_0$

The time between arrival of waves is

$$\Delta t_0 = \frac{\lambda_0}{c} \text{ or } \lambda_0 = c\Delta t_0$$

Now, let the source move towards the observer with speed $v$. The observer sees a somewhat reduced $\lambda$

$$\lambda = c\Delta t - v\Delta t$$



$\Delta t_0$ is the proper time (time in rest frame of source) between emission of waves.

Time dilation: $\Delta t = \gamma\Delta t_0 = \dfrac{\Delta t_0}{\sqrt{1-v^2/c^2}}$

$$\lambda = (c-v)\Delta t = (c-v)\frac{\Delta t_0}{\sqrt{1-v^2/c^2}} = \frac{(c-v)}{\sqrt{c^2-v^2}}\lambda_0$$

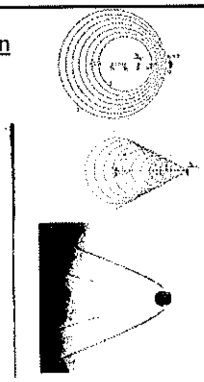
or $$\lambda = \lambda_0\sqrt{\frac{c-v}{c+v}}$$ BLUESHIFT

If the source and the observer are moving away from each other, $v \rightarrow -v$ and

$$\lambda = \lambda_0\sqrt{\frac{c+v}{c-v}}$$ REDSHIFT

## Čerenkov Radiation

$v > c/n$





## Apparent Paradoxes

**The twin paradox:**

Identical twins, Pat and Mike, born at the same moment

At age 20: Pat travels to α-Centauri at a speed of $0.9c$ and then returns to Earth at the same speed.
Mike remains on Earth and works the family farm.

Pat returns to Earth to find that *8.4* years have passed.
i.e Mike is *28.4* years of age.

Pat however has only aged

$$t = t_p \gamma = 3.7 \text{ years}$$

i.e. Pat is *23.7* years of age.



**OK, that's the picture in Mike's frame.**

**What happens in Pat's frame?**

Pat will see himself at rest and Mike traveling at $0.9c$

Doesn't this mean that at the reunion their ages will be reversed? **PARADOX**

**Symmetry in what the twins observe is only apparent .**

Symmetry is broken by the out and back nature of Pat's journey.

Only Mike is always in a single inertial frame. Pat is in several



Proper application of Lorentz transforms → Pat is indeed younger

One can also invoke General Relativity since accelerations are involved.

Thornton/Rex, Modern Physics for Scientists and Engineers, 2/e
Table 2.1

**TABLE 2.1**
**Twin Paradox Analysis***

| Item | Measured by Frank (remains on Earth) | Measured by Mary (traveling astronaut) |
|---|---|---|
| Time of total trip | $T = 2L/v$ | $T' = 2L/\gamma v$ |
| Total number of signals sent | $fT = 2fL/v$ | $fT' = 2fL/\gamma v$ |
| Frequency of signals received at beginning of trip $f'$ | $f\sqrt{\frac{1-\beta}{1+\beta}}$ | $f\sqrt{\frac{1-\beta}{1+\beta}}$ |
| Time of detecting Mary's turnaround | $t_1 = L/v + L/c$ | $t_1' = L/\gamma v$ |
| Number of signals received at the rate $f'$ | $f't_1 = \frac{fL}{v}\sqrt{1-\beta^2}$ | $f't_1' = \frac{fL}{v}(1-\beta)$ |
| Time for remainder of trip | $t_2 = L/v - L/c$ | $t_2' = L/\gamma v$ |
| Frequency of signals received at end of trip $f''$ | $f\sqrt{\frac{1+\beta}{1-\beta}}$ | $f\sqrt{\frac{1+\beta}{1-\beta}}$ |
| Number of signals received at rate $f''$ | $f''t_2 = \frac{fL}{v}\sqrt{1-\beta^2}$ | $f''t_2' = \frac{fL}{v}(1+\beta)$ |
| Total number of signals received | $2fL/\gamma v$ | $2fL/v$ |
| Conclusion as to other twin's measure of time taken | $T' = 2L/\gamma v$ | $T = 2L/v$ |

*After A. French, Special Relativity, New York: W. W. Norton, 1968, p. 158.



## The Pole in the Barn Paradox:

Farmer Brown wants to store a pole in his barn but the pole is too long.


He hires an out of work runner (disqualified for use of banned substances) to carry the pole into the barn at a speed of $0.9c$

$$\beta = \frac{v}{c} = 0.9$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.29$$

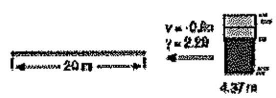


In the barn's (or the farmer's) frame, $L = L_0 / \gamma =$ 8.73 m, pole should momentarily fit.


This is the barn frame.



In the runner's frame, the barn appears to have shrunk to $L = L_0 / \gamma$ = 4.37 m, the pole should stick out even worse than before




**PARADOX**

Does it fit or not?

Q. What is meant by "the pole is in the barn"?

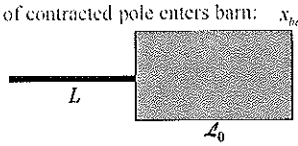
A. Momentarily, both the front and the back doors of the barn can be closed containing the entire pole.

**The closing of the doors, if simultaneous in one frame, need not be so in another frame.**

Let's consider events involving the two ends of the pole.

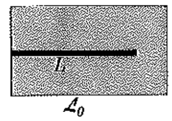
Barn frame:

Front of contracted pole enters barn: $x_{barn} = 0,\ t_{barn} = 0$

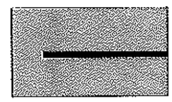





Back of contracted pole enters barn: $t_{barn} = \dfrac{8.73\ \text{m}}{0.9c} = 32.29\ \text{ns}$




Front of contracted pole leaves barn: $t_{barn} = \dfrac{10\ \text{m}}{0.9c} = 37.04\ \text{ns}$

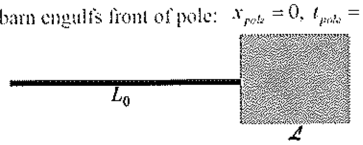


Back of pole enters barn before front leaves. Doors can be simultaneously shut momentarily. Let's quickly close and open the doors at $t_{barn}$= 32.35 ns

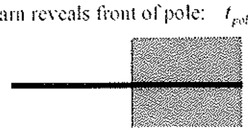


Pole frame:

Front of contracted barn engulfs front of pole: $x_{pole} = 0,\ t_{pole} = 0$

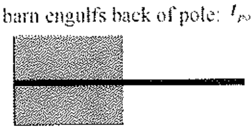



Back of contracted barn reveals front of pole: $t_{pole} = \dfrac{4.37\ \text{m}}{0.9c} = 16.4\ \text{ns}$





Front of contracted barn engulfs back of pole: $t_{pole} = \dfrac{20\ \text{m}}{0.9c} = 74.07\ \text{ns}$



Back of contracted barn reveals back of pole: $t_{pole} = 16.14\ \text{ns} + 74.07\ \text{ns} = 90.21\ \text{ns}$

In the barn frame both doors close and immediately reopen at $t_{barn}$= 32.35 ns.

Let's Lorentz transform to the pole frame.

Front door: $t_{pole} = \gamma\left(t_{barn} - \frac{v x_{barn}}{c^2}\right) = 2.29(32.35\text{ ns} + 0) = 74.08\text{ ns}$

Back door: $t_{pole} = \gamma\left(t_{barn} - \frac{v x_{barn}}{c^2}\right) = 2.29\left(32.35\text{ ns} - \frac{(0.9c)(10\text{m})}{c^2}\right)$

$= 5.38\text{ ns}$

Doors closing-opening are simultaneous in the barn frame but not in the pole frame.

From the pole point of view the front door closes and opens just after it passes the back of the pole. The back door closes and opens earlier before the front of the pole is reached. The doors closings and openings are not simultaneous and allow the passage of the barn over the pole without hitting either door



**So what finally happens? New element. What happens as the pole is brought to stop?**

Suppose the farmer grabs the back end of the pole as it enters the barn and brings the pole to a stop.

The pole must then revert to its proper length in the barn frame. Either the door is knocked back open, a hole is punched in the far wall of the barn or both.

Farmer Brown is not happy!

$K = mc^2(\gamma - 1) = (25\text{ kg})(3\times 10^8\text{ m/s})^2(2.29-1) = 3\times 10^{18}\text{ J}$

Hiroshima, August 1945, *Little Boy*, $\approx$ 15 kilotons $= 6.3\times 10^{13}$ J



## 4-D Minkowski Spacetime

**Recall the matrix formulation of the Lorentz transform for a boost along the (+) *x*-axis**

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\left(\frac{-v}{c^2}\right) & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$ct' \leftarrow t'$ $\quad$ $t \rightarrow ct'$

An event location is noted as a 4-D vector with the 4th dimension being time.

To keep the same units for all axes replace $t$ with $ct$.



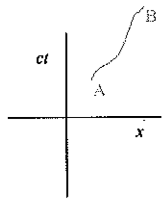
Hermann Minkowski

1864-1909

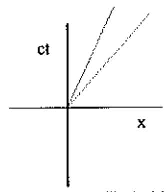
"The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality."

**Hermann Minkowski in his talk at the 80th Assembly of German Natural Scientists and Physicians, September 21, 1908**

## Minkowski space and Minkowski diagrams



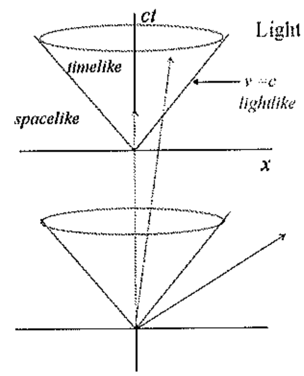

Events A and B are connected by a trajectory called a **worldline**




-Spaceship leaves origin w. velocity $v$. *Slope=c/v*

- Light signal velocity $c$






Light cone

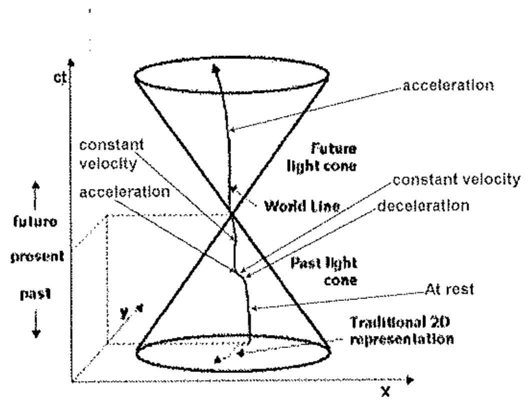
**Points or locations are events.**

**Events move on paths called** world lines

Only timelike world lines are allowed

Cannot see outside light cone








<u>Lorentz Invariants and4-dimensional distance</u>: *interval* ($s$)

In 3-D: $d^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is the square of the distance between 2 points. $s = 0$ only if the two points are colocated.

Galilean transforms $\Rightarrow d^2$ and hence $d$ are the same in any inertial frame and are thus said to be *invariant.*

Is there a quantity, similar to $d^2$, which is invariant under the Lorentz transforms in 4-D?

Consider two inertial frames, F and F', and the quantities

$$s^2 = x^2 - (ct)^2$$

$$s'^2 = x'^2 - (ct')^2$$

Applying the Lorentz transforms we find that $s^2 = s'^2$, so the quantity $s^2$ is an invariant.

Including the other spatial coordinates, $y$ and $z$, we have $s^2 = x^2 + y^2 + z^2 - (ct)^2$ as an invariant.

In analogy to the distance between two points in 3-D we can determine the 4-D separation of two events in Minkowski space.

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (z)^2 - (\Delta ct)^2$$

$\Delta s$ is called the *spacetime interval* between events