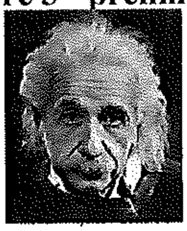
University of San Francisco

Modern Physics for Frommies I

Albert Einstein's Universe

Lecture 3 - preliminary



Agenda

- Administrative matters
  - International Year of Astronomy (400 yrs. since Galileo) at UC Berkeley
  - More about Albert Michelson
  - Upcoming Physics Colloquia
- Lorentz Transformations
- Time Dilation
- Lorentz Contraction

Stars of stargazing will speak at UC Berkeley

University launches free monthly lectures during International Year of Astronomy

By Susanna Bohan

Berkeley Voice

22 January 2009

astro.berkeley.edu/iya

Einstein's Postulates

(1) Relativity Principle:

The laws of physics have the same form in all inertial reference frames.

Inertial frame ≡ one in which Newton's laws are valid, i.e. one in which an object subject to no external force moves in a straight line with constant velocity.

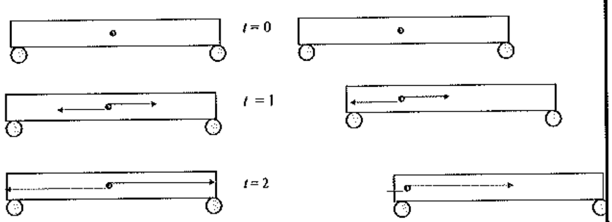
(2) Constancy of the speed of light:

Observers in all inertial frames measure the same value for the speed of light in a vacuum.

Light propagates through empty space with a definite speed, c, independent of the velocity of source or observer.

Relative Simultaneity:

Now consider just one moving train car with a light flasher mounted at the center.



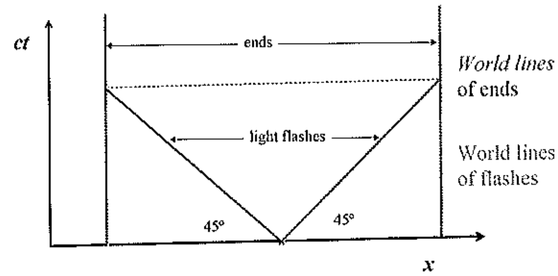
Observer on train.

Light flashes arrive front and back simultaneously.

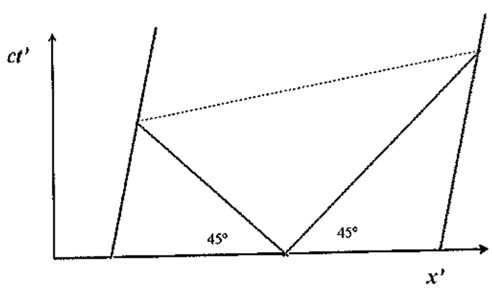
Observer on platform.

Arrival of flashes is not simultaneous.

Spacetime Diagrams



Observer on the Train



Observer on the Platform

The Lorentz Transformation

Event = displacement from origin ≡ location + time

A 4-D vector using column matrix notation

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}$$

Change frames via a transform(ation).

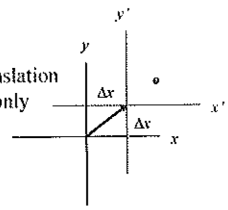
In Galilean relativity we need only deal with the 3-D vector

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Time is an absolute

Transformations between stationary (w.r.t. each other) frames

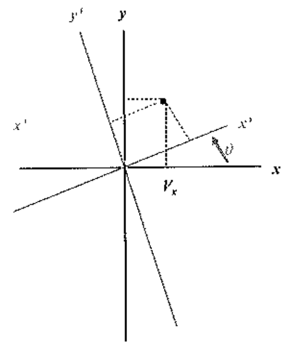
Translation only



$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \text{Transform} \rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$x' = x - \Delta x$$
$$y' = y - \Delta y$$
$$z' = z \pm \Delta z$$

Stationary frames rotation only

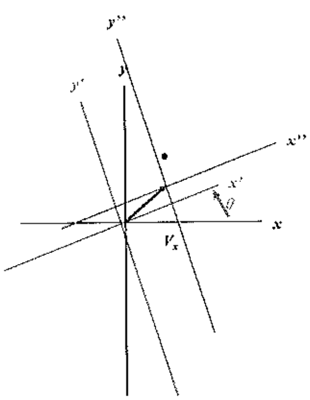


$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

In 3-D this becomes rather messy[1].

Rotation and Translation: Composed of two successive transforms.

1. H. Goldstein, *Classical Mechanics*, Addison-Wesley (1950) p. 109



Transforms are linear and orthogonal

Continuous series of events ≡ *trajectory*

If the events are locations the trajectory is called a *world line.*

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

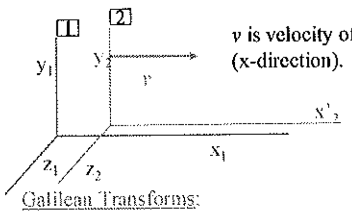
Similarly for y and z

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

Transformations between stationary frames do not change v or a, they are *invariants.*

Now, consider two inertial frames moving relative to one another.

One dimension, say x. Note that no matter what the direction of the velocity between frames we can always perform an ordinary spatial rotation to align the x-axis with that velocity.



v is velocity of frame 2 w.r.t. frame 1 (x-direction). v = const. ≡ *boost*

Galilean Transforms:

$$x_2 = x_1 - vt \qquad x_1 = x_2 + vt \qquad v_2 = v_1 - v$$

$$y_2 = y_1 \qquad v_1 = v_2 + v$$

$$z_2 = z_1 \qquad t_1 = t_2$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Failures of Galilean Relativity:
Maxwell's equations are not invariant
Requires preferred reference frames which were ruled out by experiments like those of Michelson and Morley.

Now, apply Einstein's postulates

Einstein's Postulates

(1) Relativity Principle:

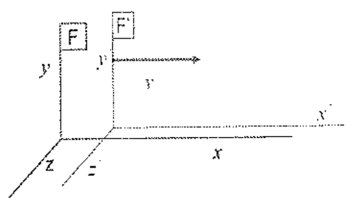
The laws of physics have the same form in all inertial reference frames.

***Inertial frame* ≡ one in which Newton's laws are valid. i.e. one in which an object subject to no external force moves in a straight line with constant velocity.**

(2) Constancy of the speed of light:

Observers in all inertial frames measure the same value for the speed of light in a vacuum.

Light propagates through empty space with a definite speed, c, independent of the velocity of source or observer.



At $t = t' = 0$ the 2 frames are coincident. A source at the origin emits a pulse of light.

Einsteins postulates⇒ Physical laws must be phrased identically in the 2 systems

(1) ⇒ A wave equation of the form

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

describes the propagation of light in both frames

(2) ⇒ c will be the same in both frames

Derive the transformation:
Since the constant interframe velocity is along x we have

$$y=y'$$
$$z=z'$$

Require a linear transformation so that events in the 2 frames are 1-1

Simplest is of the form

$$x' = \gamma (x - vt)$$

The inverse transformation is

$$x = \gamma' (x' + vt')$$

Where postulate (1) requires that $\gamma = \gamma'$

Since c is the same in the 2 frames we must have

$$ct' = \gamma (ct - vt)$$

and

$$ct = \gamma (ct' + vt')$$

Divide both equations by c

$$t' = \gamma t (1 - v/c)$$
$$t = \gamma t' (1 + v/c)$$

Substituting for t $\quad t' = \gamma^2 t' (1 - v/c)(1 + v/c)$

but $\quad t' = t$

$$\Rightarrow \quad \gamma^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$ is called the *Lorentz factor*.

$$\beta = \frac{v}{c}$$ is called the *speed*

Returning to the transforms for $x \leftrightarrow x'$

$$x' = \gamma (x - vt)$$
$$x = \gamma (x' + vt')$$

Substituting for x'

$$x = \gamma^2 (x - vt) + \gamma v t'$$

And solving for t'

$$t' = \frac{x}{\gamma v}\left(1-\gamma^2\right)+\gamma t$$

Note that

$$\frac{1-\gamma^2}{\gamma v} = \frac{1-\frac{1}{1-\beta^2}}{\frac{1}{\sqrt{1-\beta^2}}v} = \frac{\frac{1-\beta^2-1}{1-\beta^2}}{\frac{1}{\sqrt{1-\beta^2}}v} = \frac{1-\beta^2-1}{\sqrt{1-\beta^2}\,v} = -\frac{\gamma\beta^2}{v}$$

Substituting into

$$t' = \frac{x}{\gamma v}\left(1-\gamma^2\right)+\gamma t$$

And manipulating

$$t' = \gamma (t - vx/c^2)$$

or the inverse

$$t = \gamma (t' + vx'/c^2)$$

In summary, the Lorentz transforms (frame boost in *x*) are:

$x' = \gamma(x - vt)$ And the inverses $x = \gamma(x' + vt)$

$y' = y$ $y = y'$

$z' = z$ $z = z'$

$t' = \gamma(t - vx/c^2)$ $t = \gamma(t' + vx'/c^2)$

Note that for v << c these reduce to the familiar Galilean transforms.

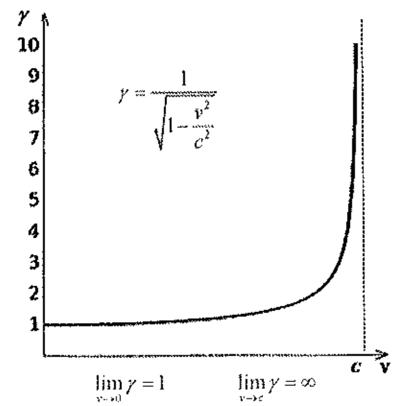
$x_2 = x_1 - vt \quad x_1 = x_2 + vt$

$y_2 = y_1$

$z_2 = z_1$

$v_2 = v_1 - v$

$v_1 = v_2 + v$



Consequences of the Lorentz Transforms

"The world turned upside down"

-An English ballad, supposedly played bythe British band at Lord Cornwallis' surrender At the siege of Yorktown (1781

Simultaneity

Time can no longer be regarded as an absolute quantity.

The time interval between 2 events and even whether 2 events are simultaneous depends on the observer's reference frame.

Simultaneous $\Rightarrow \Delta t = t_1 - t_2 = 0$

But

$\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$

Theorem: If 2 events are simultaneous and colocated in frame F they are simultaneous and colocated in F'.

Thus simultaneity is not an absolute concept but is relative.

Can we Lorentz transform to a frame in which the order of events is reversed?

Consider two events

$$\Delta t = t_1 - t_2$$

$$\Delta t' = t_1' - t_2'$$

$$\Delta t' = \gamma(\Delta t - v\Delta x/c^2)$$

Let $\Delta t > 0$ then, $\Delta t' < 0$ if $v\Delta x/c^2 > \Delta t$

$$\frac{\Delta x}{\Delta t} > \frac{c^2}{v} > \frac{c^2}{c} = c$$

NOT ALLOWED

There are metaphysical arguments involving causality, predestination, free will etc.

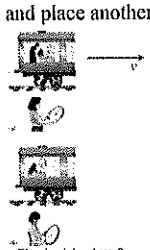
The superluminal murder trial:

Time Dilation

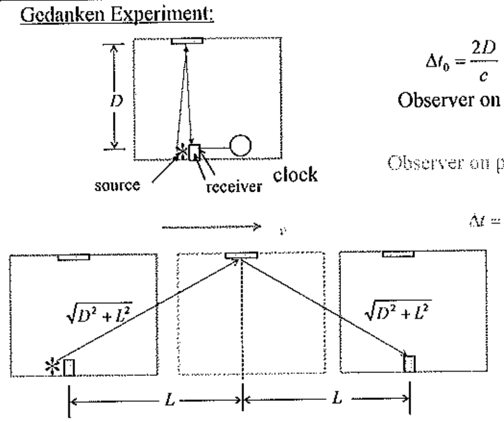
Time passes differently in different reference frames. In fact, clock's moving relative to an observer are measured by that observer to run more slowly than clocks at rest.

Let Δt_0 be the time interval measured by a clock at rest w.r.t an observer O_1. Δt_0 is called the proper time.

Let's use train cars again and place another observer, O_2, on the platform



Gedanken Experiment:



$$\Delta t_0 = \frac{2D}{c}$$

Observer on train

Observer on platform

$$\Delta t = \frac{2\sqrt{D^2+L^2}}{c}$$

Speed of light is c in all inertial frames

$$c = \frac{2\sqrt{D^2 + L^2}}{\Delta t} = \frac{2\sqrt{D^2 + \frac{v^2(\Delta t)^2}{4}}}{\Delta t}$$

Square both sides and solve for Δt

$$c^2 = \frac{4D^2}{(\Delta t)^2} + v^2$$

$$\Delta t = \frac{2D}{c\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{recall that } \frac{2D}{c} = \Delta t_0$$

$$\underline{\Delta t = \gamma \Delta t_0}$$

Lorentz Transforms:

Once again, we use the proper time Δt_0 as the time interval in the trainframe where the clock is at rest w. r. t. the observer.

$$\Delta t_0 = t_2 - t_1$$

Go via Lorentz transform to the frame in which the clock is moving with velocity v w.r.t the observer

$$\Delta t = \frac{(t_2 - t_1) - (v^2/c^2)(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

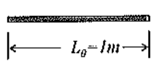
Clock is fixed in the train system, so $x_2 - x_1 = 0$ and

$$\underline{\Delta t = \gamma \Delta t_0}$$

Length Contraction

The length of an object is measured to be shorter when it is moving w.r.t. an observer than when it is at rest.

Observer is at rest w.r.t meter stick.

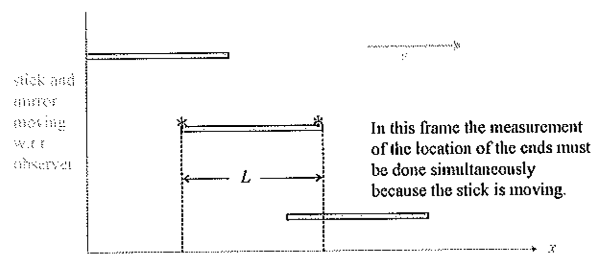


L_0 is called the *proper length*

☺ O_1

Measurements of the locations of the 2 ends do not have to be made simultaneously since the stick is at rest w.r.t. O_1

Now, let's look at a 2nd observer with the stick passing at velocity v.



☺ O_2

$$x_0 = \gamma(x - vt) \Rightarrow L_0 = \gamma(L - v\Delta t)$$

Lab frame: $\Delta t = 0 \Rightarrow L_0 = \gamma L$

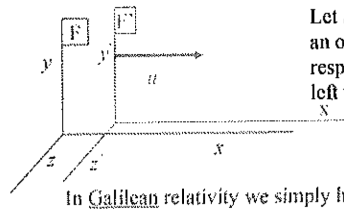
$$\underline{L = \frac{1}{\gamma} L_0} \qquad \textit{Lorentz contraction}$$

$$L = \frac{1}{\gamma} L_0 = \sqrt{1 - \frac{v^2}{c^2}} L_0$$

Lorentz and Fitzgerald (1890's): Length contracts by a factor of $(1-v^2/c^2)^{1/2}$ in the direction of motion through the ether, an electrical effect on intermolecular forces.

Rather than invoking an unknown electrical interaction with intermolecular forces we see the contraction arising from a coordinate transform under which the speed of light is invariant.

Velocity Addition



Let u and u' be the velocities of an object in F and F' respectively. F' moves to the left with velocity v w.r.t. F

In Galilean relativity we simply have $v' = v - u$

and

$v = v' + u$

This is not in agreement with the Lorentz transforms

$x' = \gamma(x - ut)$	And the inverses	$x = \gamma(x' + ut)$
$y' = y$		$y = y'$
$z' = z$		$z = z'$
$t' = \gamma(t - ux/c^2)$		$t = \gamma(t' + ux'/c^2)$
$v_{x'} = \frac{\Delta x'}{\Delta t'}$		$v_x = \frac{\Delta x}{\Delta t}$

Using differential calculus or else some messy algebra

$$v_{x'} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \qquad v_x = \frac{v_{x'} + u}{1 + \frac{uv_{x'}}{c^2}}$$

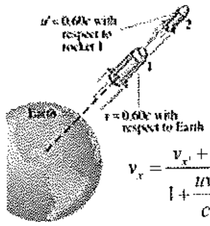
$$v_{y'} = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad v_y = \frac{v_{y'}}{\gamma\left(1 + \frac{uv_{x'}}{c^2}\right)}$$

$$v_{z'} = \frac{v_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad v_z = \frac{v_{z'}}{\gamma\left(1 + \frac{uv_x}{c^2}\right)}$$

Note the effects transverse to the boost due to the Lorentz transformation of the time coordinate.

Again, for v and $u << c$ these reduce to the Galilean velocity transforms

Example: Velocity Addition



Find speed of 2 w.r.t. Earth

Earth is frame F.
2 is frame F'. x, x' are along flight

$$v_x = \frac{v_{x'} + u}{1 + \frac{u v_{x'}}{c^2}} = \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c$$

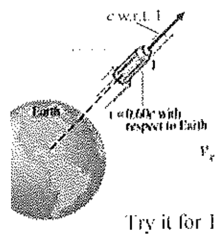
Galilean transform $\rightarrow v_x = 1.20c$

There is no addition of velocities that will result in $v > c$

TRY IT

Example: Constancy of c

Replace 2 with light from 1's headlight in previous example



Find speed of light w.r.t. Earth

Earth is frame F.
1 is frame F'. x, x' are along flight

$$v_x = \frac{v_{x'} + u}{1 + \frac{u v_{x'}}{c^2}} = \frac{c + 0.60c}{1 + \frac{(0.60c)c}{c^2}} = \frac{1.60c}{1.60} = c$$

Try it for 1 traveling at $0.99c$ w.r.t. Earth

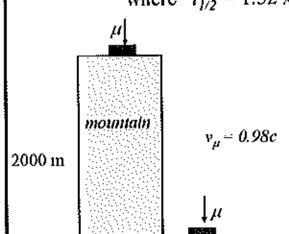
$$v_x = \frac{v_{x'} + u}{1 + \frac{u v_{x'}}{c^2}} = \frac{c + 0.99c}{1 + \frac{(0.99c)c}{c^2}} = \frac{1.99c}{1.99} = c$$

Experiments and observations

Muon decay: $\mu^{\pm} \rightarrow e^{\pm} \nu_e \nu_\mu$

Radioactive decay law: $N = N_0 e^{-(\ln 2)\left(\frac{t}{t_{1/2}}\right)}$

where $t_{1/2} = 1.52 \times 10^{-6}$ seconds



Count for some period of time

Top – 1000 ± 31.6 μ

Bottom = 540 ± 23.2 μ

Classically, a particle moving at $0.98c$ covers 2000 m in 6.8×10^{-6} sec.

Decay law $\Rightarrow$ only 45 μ should survive the trip

Relativistically, we realize that the quoted half-life of 1.52×10^{-6} sec. is that of a μ at rest. At $\beta = 0.98$, time dilation is significant.

An observer in the lab will perceive that a clock moving with the m to be slowed by a factor of g.

$\beta = 0.98 \Rightarrow \gamma = 5$

Decay law $\Rightarrow$ 538 μ should survive the trip. Agreement with experiment.

Alternatively, examine the problem from the point of view of an observer traveling with the μ.

This observer sees the 2000 m flight path as length contracted by a factor of $1/\gamma$ to 400 m.

The time to travel this contracted difference is thus reduced by a factor of 1/5.

Decay law $\Rightarrow$ *538* surviving μs.

Identical result, in agreement with experiment, is obtained by using either time dilation or space contraction.